**1)**

P => Q = ~P ∨ ~Q

Q => ~P = ~Q ∨ ~P

|  |  |  |  |
| --- | --- | --- | --- |
| **P** | **Q** | **P => ~Q** | **Q => ~P** |
| T | T | F | F |
| T | F | T | T |
| F | T | T | T |
| F | F | T | T |

P ⬄ ~Q = (P => ~Q)∧ (~Q => P)

= (~P ∨ ~Q)∧(Q ∨ P)

|  |  |  |  |
| --- | --- | --- | --- |
| **P** | **Q** | **P ⬄ Q** | **((P∧~Q)∨(~P∧Q))** |
| T | T | F | F |
| T | F | T | T |
| F | T | F | F |
| F | F | T | T |

**2)**

(S => F) => (~S => ~F)

= (~S ∨ F) => (S ∨~F)

= ~(~S∨F) ∨(S∨~F)

= (S∧~F) ∨(S∨~F)

|  |  |  |
| --- | --- | --- |
| **S** | **F** | **(S∧~F)∨(S∨~F)** |
| T | T | T |
| T | F | T |
| F | T | F |
| F | F | T |

**The sentence is neither valid nor unsatisfiable since it is true for some worlds, but not all.**

(S => F) => ((S∨H) => F)

= (~S∨F) => (~(S∨H)∨F)

= ~(~S∨F)∨((~S∧~H)∨F)

= (S∧~F)∨((~S∧~H)∨F)

|  |  |  |  |
| --- | --- | --- | --- |
| **S** | **H** | **F** | **(S∧~F)∨((~S∧~H)∨F)** |
| T | T | T | T |
| T | T | F | T |
| T | F | T | T |
| T | F | F | T |
| F | T | T | T |
| F | T | F | F |
| F | F | T | T |
| F | F | F | T |

**The sentence is neither valid nor unsatisfiable since it is true for some worlds, but not all.**

((S∧H)=>F)⬄((S=>F)∨(H=>F))

=(~(S∧H)∨F)⬄((~S∨F)∨(~H∨F))

=((~S∨~H)∨F)⬄(~S∨~H∨F)

=[(~S∨~H∨F)=>(~S∨~H∨F)]∧[ (~S∨~H∨F)=>(~S∨~H∨F)

=(~S∨~H∨F)=>(~S∨~H∨F)

=~(~S∨~H∨F)∨(~S∨~H∨F)

=(S∧H∧~F)∨(~S∨~H∨F)

|  |  |  |  |
| --- | --- | --- | --- |
| **S** | **H** | **F** | **(S∧H∧~F)∨(~S∨~H∨F)** |
| T | T | T | T |
| T | T | F | T |
| T | F | T | T |
| T | F | F | T |
| F | T | T | T |
| F | T | F | T |
| F | F | T | T |
| F | F | F | T |

This sentence is valid since it is true for all worlds.

**3)**

Variables:

I – Immortal

M – Mythical

A – Mammal

H – Horned

G – Magical

Knowledge Base:

M => I

~M => (~I ^ A)

(I v A) => H

H => M

M => I

= **~M v I**

~M => (~I ^ A)

= M v (~I ^ A)

= **(M v ~I) ^ (M v A)**

(I v A) => H

= ~(I v A) v H

= (~I ^ ~A) v H

= **(H v ~I) ^ (H v ~A)**

H => G

= **~H v G**

CNF:

(~M v I) ^ (M v ~I) ^ (M v A) ^ (H v ~I) ^ (H v ~A) ^ (~H v G)

Prove Mythical (M):

**1 ~M v I**

**2 M v ~I**

**3 M v A**

**4 H v ~I**

**5 H v ~A**

**6 ~H v G**

**7 ~M to prove contradiction**

**---------------------------**

8 ~I 2 & 7

9 A 3 & 7

10 H 5 & 9

11 G 6 & 10

12 ~M v H 1 & 4

Cannot prove contradiction, therefore the unicorn is not always mythical.

Prove Magical (G):

**1 ~M v I**

**2 M v ~I**

**3 M v A**

**4 H v ~I**

**5 H v ~A**

**6 ~H v G**

**7 ~G to prove contradiction**

**---------------------------**

8 ~H 6 & 7

9 ~A 5 & 8

10 ~I 4 & 8

11 M 3 & 9

**12 I 1 & 11 🡨 Contradiction with 10!**

Since we found a contradiction, we know that unicorns are magical.

Prove Horned (H):

**1 ~M v I**

**2 M v ~I**

**3 M v A**

**4 H v ~I**

**5 H v ~A**

**6 ~H v G**

**7 ~H to prove contradiction**

**---------------------------**

8 ~A 5 & 7

9 M 3 & 8

10 I 1 & 9

11 H 4 & 10 🡨 Contradiction with 7!

Since we found a contradiction, we know that unicorns are horned.